International Journal of Heat and Mass Transfer 52 (2009) 2132–2139

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Modeling and simulation of heat transfer phenomena during investment casting

M.M.A. Rafique [a,*], J. Iqbal [b]

[a] Hamburg University of Technology, M-11, Denickestrasse 15, Hamburg 21073, Germany
[b] Department of Metallurgical and Materials Engineering, University of Engineering and Technology, G. T. Road, Lahore, Pakistan

ARTICLE INFO

Article history:
Received 29 February 2008
Received in revised form 10 November 2008
Available online 26 December 2008

Keywords:
Investment casting
Shrinkage
Heat transfer coefficients
Silica
Zircon

ABSTRACT

Determining the heat transfer phenomena during casting processes is an important parameter for measuring the overall performance of process. It gives information about the properties of the metal being casted and its possible behavior in the mold during casting process. Improper determination of heat transfer phenomena and use of improper molding materials and casting conditions leads to defects such as misruns, cold shuts, shrinkage, pin holes, air holes and porosity in final product. A mathematical model was developed using standard transport equations incorporating all heat transfer coefficients to calculate the time for solidification of metal in casting and computer simulation of the model was carried out in C++ to validate the model. The metal used was pure iron casted in investment molds of silica sand with zircon coating. It was shown that airflow near the mold surfaces was partially restricted due to geometry of the molds and arrangement of the pieces around a tree. So, the changes in heat transfer coefficient also contribute towards time of solidification. The time calculated was found to be in good agreement with experimental values.

1. Introduction

The investment casting process consists of producing wax patterns by injection of wax into metal dies. These patterns are attached to wax coated rod(s) to make wax pattern trees. These trees are successively dipped in ceramic slurries and sprinkled with sand to form ceramic shells. Ceramic shells are heated to high temperature in ovens to extract out the wax (dewaxing of ceramic shells). After this, these shells are fired in furnaces to sinter them and finally metal is poured in them to make castings [9]. The casted part is exact translation of wax pattern into metal part. The dimensional difference between the wax pattern produced and metal part casted occur as a result of solidification and deformation behavior of metal, wax and shell molding materials. The differences between wax pattern and die and between final part and shell are known as shrinkage allowances of wax and metal, respectively. These allowances should be taken into account before the designing of successful investment casting system.

Time taken for solidification is predominantly one of the most important factors governing the overall quality of casting and is in direct relation with shrinkage and liquid to solid state transformation of alloy in mold cavity [9]. As a general rule, the faster the solidification time, better would be strength, homogeneity, integrity and quality of casting and vice versa. The time of solidification can be determined by a combination of heat transfer and deformation analysis during the pouring and solidification of alloy in mold cavity [8]. The accuracy of the time determination results depends on heat transfer results. In this paper, the numerical simulation of heat transfer phenomena during the investment casting process is discussed to determine the time of solidification.

The shell molds were considered as a packed bed of sintered ceramic particles of different sizes and shapes. The semi-transparent effects in the silica at high temperatures, which enhance the heat transfer ability of sand to a certain extent, were incorporated by a temperature-dependent thermal conductivity in numerical modeling and simulation [14]. For shell molds made of silica with a zircon coat, data on relevant properties were given in Sabau and Viswanathan [6] and Pehlke and Jayarajan [14]. Previously, in most studies on numerical simulation of casting processes [1–4], a set of constant heat transfer coefficients are used to determine the heat flow from metal to mold. Whereas in actual situation, heat transfer coefficients (HTCs) change continuously during casting due to change in casting conditions (heating of air, mold, etc.) [8]. All these studies suffered from not incorporating these change of conditions in casting process, which inevitably affect the rate of heat transfer and thus solidification time. Anderson et al. [5], however, found that the use of single HTC for all mold surfaces was not accurate. The major contribution of present study is to determine HTCs based upon changing conditions of system as well as incorporating incremental changes in HTCs during solidification back into the initial calculations using iterative approach in programming. This approach gives better estimates of actual time calculations based upon mold and ambient conditions, which keeps on changing as

---

* Corresponding author. Tel.: +49 40 428783238; fax: +49 40 428782002.
E-mail address: ali.rafique@hotmail.com (M.M.A. Rafique).

0017-9310/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2008.11.007

Nomenclature

A	area of heated mold surface
A_T	area of top surface of mold
Cp	specific heat of metal ($\mathrm{J\,K^{-1}\,kg^{-1}}$)
g	acceleration due to gravity ($\mathrm{m\,s^{-2}}$)
Gr	Grashof number (dimensionless quantity)
h	heat transfer coefficients for free convection ($\mathrm{W\,m^{-2}\,K^{-1}}$)
h_r	heat transfer coefficients for radiation ($\mathrm{W\,m^{-2}\,K^{-1}}$)
Hf	heat of fusion ($\mathrm{J\,kg^{-1}}$)
k	thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)
L	characteristic length
m	mass of the metal
Nu	Nusselt number (dimensionless quantity)
p	perimeter
Pr	Prandtl number (dimensionless quantity)
Q_t	total quantity of heat (J)
Q_T	total rate of heat transfer (W)
R_t	thermal resistance of the mold ($\mathrm{K\,W^{-1}}$)
t	total time of solidification
$\overline{T}$	temperature for the determination of heat transfer coefficients
T_m	metal temperature
T_p	inside temperature of the mold
T_r	room temperature
T_s	surface temperature of the mold
T_∞	mold temperature

Greek symbols

β	reciprocal of film temperature ($\mathrm{K^{-1}}$)
ε	emissivity (dimensionless quantity)
ρ	density ($\mathrm{kg\,m^{-3}}$)
μ	viscosity ($\mathrm{kg\,m^{-1}\,s^{-1}}$)
σ	Stefan–Boltzmann constant ($5.670 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$)

the solidification front progresses during casting. The cooling conditions at shell mold surfaces were determined by the airflow pattern of natural convection and heat dispensing pattern of radiation. Some shell surfaces were fully open to natural convection, while others were partially open while still other were restricted as the natural convection was affected by the proximity and geometry of nearby surfaces. Thus, the heat transfer conditions at each mold surface depended on the surface length scale, configuration and surface position in the casting tree [8]. Shell emissivity was determined from experimental data [6,14]. Temperature of the shell was measured using different thermocouple arrangements around tree.

A model was developed and computer simulations of model were performed in C++ using iterative approach on constitutive equations for the heat transfer coefficients based on correlations developed for conduction, convection and radiation [12]. Castings were poured at two independent foundries, Steel Castings (Pvt.) Ltd., Gujranwala, and at Foundry Lab, University of Engineering and Technology, Lahore, in order provide sufficient experimental data and check consistency. Computer simulation results agree well with experimental data, validating the computer model. Finally, a computer simulation model was recommended and presented for calculating the time of solidification and associated heat transfer phenomena of metals and alloys.

2. Mathematical formulation

To formulate a generalized mathematical model corresponding to heat transfer phenomena during solidification of metals and alloys in molds, following assumptions were made:

a. Mold is made up of one unit consisting of one central runner (downsprue) and second associated tree geometry.
b. Mold is heated to higher temperature (pouring temperature of metal) at once.
c. Thermal properties of mold are not constant and do change with change of temperature.
d. Thermal conductivity of mold is very very low.
e. Major portion of heat is lost through open atmosphere.

Based upon preceding assumptions, following is considered:

a. Downsprue and cubical pieces attached to it are considered similar in configuration to an open ladle in which, mode of heat transfer is convection and radiation from top and conduction through walls.

2.1. Heat transfer for investment casting shell

In order to formulate a mathematical model for heat transfer problem during casting, consider superheated liquid metal poured in an investment casting shell. There are two steps in which metal losses its heat,

a. First step in which metal losses its sensible heat (superheat).
b. Second step during which metal losses its heat of fusion (Hf).

Time of solidification is determined separately during each interval using different heat transfer coefficients (HTCs) and then individual times are summed up together to get total time taken for solidification.

2.1.1. First step of heat transfer

When metal is poured in investment casting shell, whole of the shell is quickly heated to high temperature (temperature of molten metal), heat transfer also starts subsequently and temperature of metal starts decreasing. As a result of this, temperature of mold starts increasing. During this step total heat transfer from top and walls is

$$Q_t = Q_T \cdot t_1 \tag{1}$$

where Q_t is the total quantity of heat lost from top and wall, Q_T is total rate of heat transfer by convection and radiation from top and conduction through walls and t_1 is the total time taken for heat transfer (time of solidification in first step).

Heat transfer occurs by three modes namely convection, conduction and radiation; total rate of heat transfer (Q_T) is summation of rate of heat transfers by each mode.

$$Q_T = Q_{T1} + Q_{T2} + Q_{T3} \tag{2}$$

where Q_{T1}, Q_{T2}, Q_{T3} are the rate of heat transfers by convection and radiation from top surface, conduction through walls and convection and radiation from heated mold surface, respectively.

Rate of heat transfer by convection and radiation from top may be written as:

$$Q_{T1} = (h + h_r)_{\overline{T}} \cdot A_T \cdot (\overline{T} - T_\infty) \tag{3}$$

where $\overline{T} = \frac{1}{2}(T_p + T_m)$, temperature at which HTCs are determined, h and h_r are the heat transfer coefficients for free convection and radiation, respectively, A_T is area of top surface of mold and T_∞ is mold temperature. Major portion of heat in this process of heat transfer is lost through top surface thus Q_{T1} is the major rate of heat transfer during solidification. This is the reason; risers of castings are usually covered by some insulating compound or bad heat-conducting medium to keep metal liquid for a longer period of time so that proper feeding of metal could be achieved [9].

Rate of heat transfer by conduction may be written as:

$$Q_{T2} = \frac{(T_p - T_\infty)}{R_t} \tag{4}$$

where Tp and T_∞ are the inside and outside temperatures of mold and R_t is the thermal resistance of mold wall.

Thermal resistance of the mold here though very high, is not constant, but keeps on changing with change of temperature and contributes towards overall effect of heat transfer [8].

Similarly, rate of heat transfer again by convection and radiation from outer heated wall of mold may be written as:

$$Q_{T3} = (h + h_r)_{T_s} \cdot A \cdot (T_s - T_\infty) \tag{5}$$

where T_s is the surface temperature of the mold, T_∞ is the outside (surrounding) temperature of the mold $[(T_\infty = T_r), T_r$ = room temperature], h and h_r are heat transfer coefficients (HTCs) for free convection and radiation, respectively, and A is area of heated mold surface towards ambient. This mode of heat transfer has very little contribution towards overall rate of heat transfer during first step, as initially surface of mold is at ambient and gets heated slowly in small increments thus starts transferring heat. This mode of heat transfer also has little contribution due to high thermal resistance of mold material(s), which stops most of heat from coming out of the mold [8].

Putting the values in Eq. (2)

$$Q_T = (h + h_r)_{\overline{T}} \cdot A_T \cdot (\overline{T} - T_\infty) + \frac{(Tp - T_\infty)}{R_t} + (h + h_r)_{Ts} \cdot A \cdot (Ts - T_\infty) \tag{6}$$

Total quantity of heat transferred (Q_t) is actually the heat lost by metal as its *sensible heat*, which may be written as [9],

$$\begin{aligned} Q_t &= mCp\Delta T \\ Q_t &= mCp(Tp - Tm) \end{aligned} \tag{7}$$

where, Cp is specific heat of metal, Tp and Tm are pouring and melting temperatures of the metal, respectively, and m is the mass of metal being poured.

Combining Eqs. (6) and (7) and putting values in Eq. (1)

$$\begin{aligned} mCp(Tp - Tm) &= (h + h_r)_{\overline{T}} \cdot A_T \cdot (\overline{T} - T_\infty) + \frac{(Tp - T_\infty)}{R_t} \\ &\quad + (h + h_r)_{Ts} \cdot A \cdot (Ts - T_\infty) \cdot t_1 \\ t_1 &= \frac{mCp(Tp - Tm)}{(h + h_r)_{\overline{T}} \cdot A_T \cdot (\overline{T} - T_\infty) + \frac{(Tp - T_\infty)}{R_t} + (h + h_r)_{Ts} \cdot A \cdot (Ts - T_\infty)} \end{aligned} \tag{8}$$

This is the expression for the calculations of time for solidification of metal during pouring and subsequent freezing in investment casting molds during step in which it losses all its *sensible heat*.

Various factors affect this time of solidification during first step such as thermal conductivity of mold material, casting conditions, pouring temperature, specific heat of metal, etc. All these should be taken into account while designing a casting process [10]. Soon after the release of all superheat of metal second step of solidification begins.

2.1.2. Second step of heat transfer

When metal looses all its sensible heat (superheat) and reaches its melting temperature, a phase transformation occurs and metal starts loosing heat as *heat of fusion*. This heat transfer continues till whole of the metal solidifies in the mold. This is second step of heat transfer [9].

Like first step of heat transfer, heat again transfers in three modes namely convection, conduction, and radiation. This occurs in a fashion very similar to first step of heat transfer (i.e. from top, walls and heated surface of mold). Thus, total heat transfer from top and walls may be written as:

$$Q_t = Q_T \cdot t_2 \tag{9}$$

where Q_t is the total quantity of heat lost from top and walls, Q_T is total rate of heat transfer by convection and radiation from top and conduction through walls and t_2 is the total time taken for heat transfer (time of solidification in second step).

Again total rate of heat transfer is the summation of rates of heat transfer by individual modes

$$Q_T = Q_{T1} + Q_{T2} + Q_{T3} \tag{10}$$

where Q_{T1}, Q_{T2}, Q_{T3} are the rate of heat transfers by convection and radiation from top surface, conduction through walls and convection and radiation from heated mold surface, respectively.

Quantities Q_{T1}, Q_{T2} and Q_{T3} may be determined by use of Eqs. (3)–(5) with the replacement of $\overline{T}$ with Tm in Eq. (3) as this is the temperature at which HTCs are determined during second step and Tp with Tm in Eq. (4) as this is the reference temperature from which heat is transferred during conduction in second step.

When values are inserted into Eq. (10) following is obtained,

$$Q_T = (h + h_r)_{T_m} \cdot A_T \cdot (Tm - T_\infty) + \frac{(Tm - T_\infty)}{R_t} + (h + h_r)_{Ts} \cdot A \cdot (T_s - T_\infty) \tag{11}$$

In second step, total quantity of heat transferred (Q_t) is actually the heat lost by metal as its heat of fusion, which may be written as [9],

$$Q_t = mHf \tag{12}$$

where Hf is heat of fusion of metal and m is the mass of the metal.

Putting values from Eqs. (11) and (12) to Eq. (9)

$$\begin{aligned} mHf &= (h + h_r)_{T_m} \cdot A_T \cdot (Tm - T_\infty) + \frac{(Tm - T_\infty)}{R_t} + (h + h_r)_{Ts} \cdot A_T \\ &\quad \cdot (Ts - T_\infty) \cdot t_2 \\ t_2 &= \frac{mHf}{(h + h_r)_{Tm} \cdot A_T \cdot (Tm - T_\infty) + \frac{(Tm - T_\infty)}{R_t} + (h + h_r)_{Ts} \cdot A_T \cdot (Ts - T_\infty)} \end{aligned} \tag{13}$$

This is the expression for the calculations of time of solidification of metal during the interval when it transforms from liquid to solid state, i.e. the time when metal losses all its *heat of fusion*.

Factors affecting time of solidification during this step are heat of fusion of metal, thermal conductivity of mold, cooling conditions outside mold, surface area of mold exposed to ambient and extent to which mold is heated during first step [10].

Finally adding Eqs. (8) and (13) yields the final expression for calculating the time of solidification during whole period from liquid to solid,

$$t = t_1 + t_2 \tag{14}$$

Prime objective in most engineering cases is to achieve a rapid rate of heat transfer as it facilitates fine grain structure in metal which imparts strength and hardness to metal/alloy. This rapid rate of heat transfer may be achieved by use of mold materials with high thermal conductivity, forced convection conditions (blowing of air on the outer surface of the mold) or rapid cooling of mold surfaces (sprinkling of water on mold surface). This rapid rate of heat transfer however, can induce brittleness in alloy along with poor impact properties. Slow rate of heat transfer on the other hand, can induce problems of segregation especially predominant in multicomponent alloys, long columnar grains and softness that are detrimental to its further applications [9]. So, in most practical conditions an optimum rate of heat transfer is desirable which should facilitate a high strength, fine-grained material with good mechanical properties. Mostly this is achieved in conjunction with post casting heat treatment [9,10].

2.2. Determinations of HTCs

Determination of coefficients of Heat transfers is one of the most important parameter in determining rates of heat transfer during solidification. These HTCs depend upon various dimensionless numbers, which are listed below,

HTC for free convection (h) is given by Nusselt number (Nu),

$$Nu = C(Gr \cdot Pr)^m \tag{15}$$

where C and m are constants [8], Pr and Gr are Prandtl and Grashof numbers, respectively, [($Gr = \frac{g\beta\rho^2(T_o - T_\infty)L^3}{\mu^2}$, g is gravitational constant, $\beta = \frac{1}{T}$, T is film temperature, ρ is density, μ is viscosity and L is characteristic length), ($Pr = \frac{C_p\mu}{k}$, C_p is heat capacity, μ is viscosity, k is thermal conductivity)] [11].

Also,

$$Nu = \frac{h \cdot L}{k} \tag{16}$$

where h is HTC for free convection, L is characteristic length ($L = \frac{A}{p}$, A is area, p is perimeter) and k is thermal conductivity of metal/alloy [8].

Combining Eqs. (15) and (16)

$$h = \frac{C(Gr \cdot Pr)^m \cdot k}{L} \tag{17}$$

This HTC will be responsible for transfer of heat from spaces open to air (such as top mold surface, surface of wall of the mold, spaces between cubes, etc.).

HTC for radiation heat transfer depends upon temperature and emissivity of molten metal and emissivity of heated mold material for both steps of heat transfer and is given by modification of Stefan–Boltzmann law of thermal radiation [8], which may be written as:

$$h_r = \varepsilon\sigma(T_1^2 + T_2^2)(T_1 + T_2) \tag{18}$$

where, ε is emissivity of molten metal and heated mold material in both forms in which heat is transferred by radiation, σ is Stefan–Boltzmann constant ($\sigma = 5.670 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$) and T_1/T_2 are the temperatures of two bodies between which heat is transferred. Clearly h_r is a strong function of temperature.

2.3. Film temperature (T_f)

Another parameter known as film temperature (T_f) is defined as the temperature at which properties of air are determined for use in calculations of HTCs, is given by [8]:

$$T_f = \frac{1}{2}(T + T_\infty) \tag{19}$$

where $T = \overline{T}$ and $T = Ts$ during the phases in which heat is transferred by free convection and radiation from top surface and surface of heated mold walls, respectively.

2.4. Determination of thermal resistance (R_t)

Thermal resistance of mold materials is another important parameter, which contributes towards overall rate of heat transfer during conduction. Though very high, it is a strong function of temperature and keeps on changing with change of temperature. Thermal resistance of composite wall made up of finite small layers of materials of different thermal resistances may be written as [12]

$$R_t = \frac{L}{kA} \tag{20}$$

where, L is the thickness of mold wall, A is surface area of mold exposed to heat and acting as thermal resistance and k is overall thermal conductivity of composite mold wall made up of different materials.

As the mold is made up of layers of refractory materials, which individually have their own thermal conductivities, thus overall thermal conductivity could be calculated only if effect of all layers is taken into account. Thus, temperature dependent expression for thermal conductivities may be written as [14]

$$k = 0.60401 - 0.76723 \times 10^{-3}T + 0.79544 \times 10^{-6}T^2 (20\text{—}30 \text{ mesh size sand}) \tag{21}$$

$$k = 0.67570 - 0.79335 \times 10^{-3}T + 0.55621 \times 10^{-6}T^2 (40\text{—}70 \text{ mesh size sand}) \tag{22}$$

$$k = 0.33271 - 0.16762 \times 10^{-3}T + 0.17580 \times 10^{-6}T^2 (70\text{—}100 \text{ mesh size sand}) \tag{23}$$

Their effect is incorporated based upon number of layers formed by each mesh sand in the final mold wall. The overall thermal conductivity is then substituted in Eq. (20) to get overall thermal resistance which the mold wall offers to the flow of heat.

It is very clear that thermal conductivities are a strong function of temperature and keeps on changing with change in temperature.

3. Simulation using object oriented technique

The model for calculating time of solidification was programmed in an object oriented programming language C++. The iterative approach was adopted in which resultant value reports back to its initial value thus generating iterative loop, which yield the results in more effective way (i.e. keeps record of changes occurring in small increments). The Borland C++ 3.1 package was used for programming. All variables were first given certain symbol and that symbol was defined in float number format as functions. These user-defined functions are then used throughout the program. Program asks about different variables and then computes the results using predefined functions and formulas in C++ window.

Time is declared as controlling function in the program upon which all values depend. Each calculated value reiterates itself using time as parameter yielding final result as a progression of small incremental results. Program first asks about an assumed value of time of solidification and then asks about values of constants, which are present in the form of data in literature [8,11]. The value of time is asked in order to calculate incremental heat losses and use the data back into equations to get actual results. Final time of solidification is determined when assumed value of time matches or becomes nearly equal to calculated value (this is the time at which metal completely solidifies). Other user input values are entered in the program to automatically generate the results (see Table 1).

A unique feature of program is that it can work for almost all types of metals and mold materials with 8 coatings (6 coatings of 20–30 mesh sand, 1 coating of 50–70 mesh sand and 1 coating of 70–100 mesh sand). It can also take the effect of mold heating into consideration while calculating time of solidification and can give behavior of castings with preheated molds. The program also displays different auxiliary information such as amounts of heat

Table 1
Model parameter values for simulation [11,12].

Parameter	Value
ρ, density of metal, kg/m^3	7800
ε, emissivity of molten metal	0.28
ε, emissivity of mold material	0.78
Cp, specific heat of metal, J/Kg/K	750
Hf, heat of fusion of metal, J/Kg	272,000
σ, Stefan–Boltzmann constant, W m^{-2} K^{-4}	5.670×10^{-8}

released by convection, conduction and radiation, heat left in metal and change of thermal conductivity of mold materials with time and temperature.

The data obtained from C++ model is plotted in the form of graphs and then those graphs are compared with graphs obtained by plotting actual thermocouple readings in experiment. They are found in good agreement.

4. Experimental

Experimental setup consists of an investment casting shell made up of silica with zircon prime coat in which twenty simple cubical shape pieces were connected to a central rod (down sprue) (Fig. 1). Shell is made up of approximately 20 mm thick composite wall consisting of three layers of sand namely 6 layers of 20–30 mesh sand, 1 layer of 50–70 mesh sand and 1 layer of 70–100 mesh sand layer (backing layer). Downsprue was 225 mm long rod of 40 mm^2 cross-section. A pouring basin was left on the top of tree to allow the molten metal to go down to the mold. Two thermocouples were connected to shell as shown (Fig. 2). One at the outer surface of cubical piece and one in between two pieces, in order to have two readings and a better representation of thermal heat transfer pattern.

Metal used was Pure Iron super heated to temperature of 1620 °C. Metal was melted in magnesia crucible in an induction furnace. Mold temperature was kept at ambient (i.e. no mold heating was done). When metal was poured in sprue it quickly goes to bottom of mold and gets distributed in cubes through ingates. It quickly started solidifying there and temperature of metal started decreasing while that of mold started increasing. Temperature was measured from pyrometer and thermocouples while time is measured from stopwatch. This process continued up to certain limit up to which metal lost all its sensible heat. After this, temperature drop became steady and further loss of heat occurred in the form of loss of heat of fusion of metal. Temperature of metal remained steady at its melting temperature from this point onwards and no drop in temperature occurred. However, metal continued to loose all its heat till all of metal get solidified. Time taken up to this point is known as total time of solidification. The readings of thermocouples and pyrometer were then plotted in the form of graphs and compared with graphs obtained from simulation of solidification to verify the validity of model.

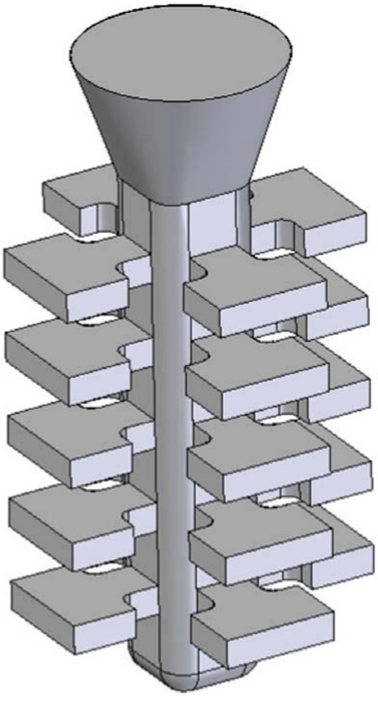

Fig. 1. Schematic of investment casting tree.

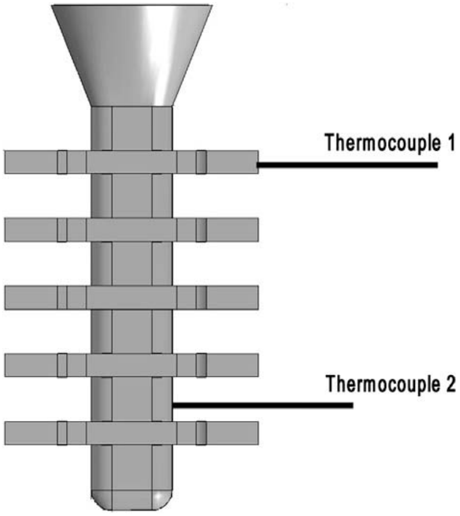

Fig. 2. Investment casting tree with thermocouples.

5. Results and discussion

Results obtained by taking readings from experiment and simulations are plotted in graphs to check the validity of model and accuracy and consistency of results. These observations primarily show time dependent behavior of heat transfer phenomena and determine the conditions of casting at different intervals of time. The readings were taken at small intervals of time to get better representation of thermal heat transfer pattern across mold surface.

Both simulation and experimental results complement each other and exhibit similar patterns of heat transfer with the passage of time till a condition is reached at which metal losses all its heat and completely solidifies. The model also shows the effect of time on different properties of mold material as well as gives heat transfer pattern across mold wall with change of time and temperature. It also takes into account the dynamic change in heat transfer coefficients with time along with change in quantities of heat(s) released as affected by change of metal and mold temperature.

5.1. Effect on metal temperature

When temperature of metal is plotted against time, a graph is obtained which is shown in Fig. 3. Fig. 3 shows a decrease in temperature of metal with the passage of time. Both readings taken from thermocouples and simulation of heat transfer phenomena show that metal temperature starts decreasing rapidly with the passage of time. This decrease is very quick in initial intervals of

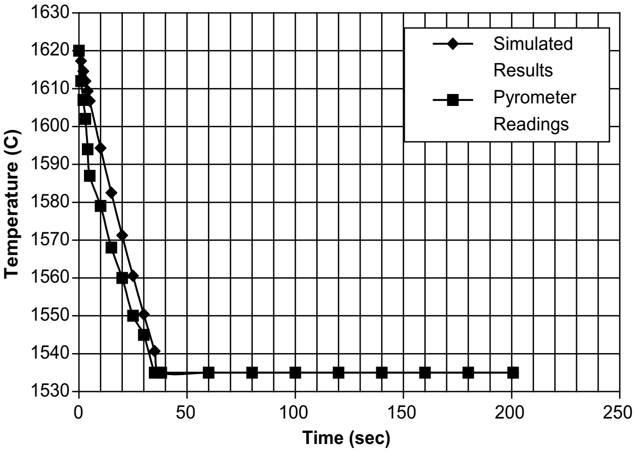

Fig. 3. Effect of time on metal temperature (simulated and measured from pyrometer).

time and becomes steady afterwards until a stage is reached at which no more decrease in temperature occurs with the passage of time. This is known as first arrest point in solidification [9]. This is the point at which metal undergoes first phase transformation. This point is very sharp in pure metals as pure metals have a sharp melting point, while in alloys this point is replaced by a range, over which alloy solidifies [13]. The curve of the graph becomes straight at this point and remains straight this point onwards. This shows that no temperature drop occur in metal after this point. This straight line continues over a period of time during which metal looses all its heat of fusion. All heat lost during this period is consumed in overcoming heat losses as a result of phase transformation. Literally all of metal solidifies completely during this period. After metal losses all its heat of fusion a further decrease in temperature occur and curve of graph goes down which continues till temperature of metal came to room temperature [9] (not shown). Curve of time and temperature obtained from readings of pyrometer have a sharper decreasing slope than that of curve of time and temperature obtained from simulation. This is because; during actual practice heat is transferred more rapidly from castings due to more severe conditions of cooling such as dissipation of heat caused by air currents, more rapid rate of heat transfer from top, etc. [12].

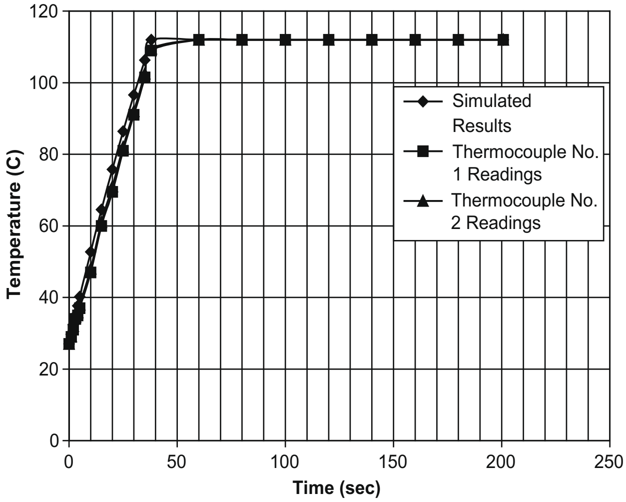

Fig. 4. Effect of time on mold outer surface temperature (simulated and measured from two thermocouples placed on mold surface).

5.2. Effect on mold temperature

When temperature readings of the thermocouples from outer surface of the mold are plotted against time graph shown in Fig. 4 is obtained. Superimposed graph (Fig. 4) shows that initially there is rapid increase in mold outer surface temperature as a result of heat transferred from hot metal poured in cold mold. As the time passes and metal looses its sensible heat, temperature of mold increases. But at a certain point when metal has exhausted all its superheat and starts giving heat of fusion, a change in slope of graph occur and graph becomes straight line on which no increase of mold temperature is depicted with passage of time. This happens because walls of the mold get heated with time, which significantly decreases their heat transfer capability. This process continues till metal looses all its heat of fusion. The time of solidification can also be obtained from the graph by noting the time till which straight line portion of graph lasts. The graph shows that actual thermocouple readings lag slightly behind the simulated readings. This happens because in actual practice there are more heat losses due to localized thermal heat transfer conditions such as air flow, access of fresh air to outer part of casting, presence of large volume of air, etc. which cause more rapid transfer of heat thus lowering of temperature, then calculated.

5.3. Effect on heat released and contained

Graphs shown in Fig. 5 are resulted when heat released from metal through the mechanisms of convection, conduction and radiation and heat contained in metal are plotted against time. Heat contents released and contained in the metal are calculated from

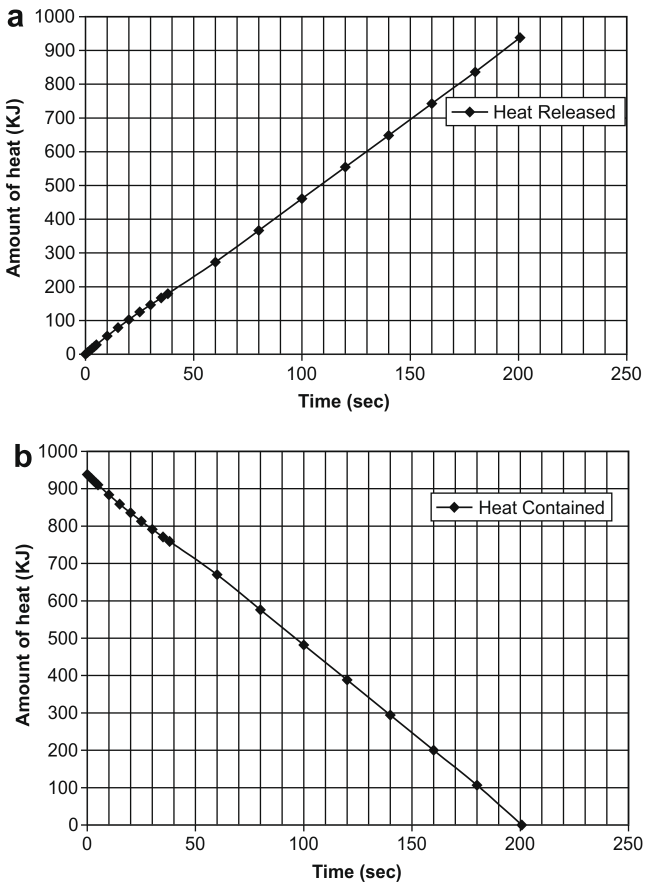

Fig. 5. Effect of time on heat contents of system. (a) Amount of heat released with time. (b) Amount of heat contained after time.

simulation in C++. Fig. 5(a) and (b) clearly shows the content of heat released by, and contained in, the metal after small intervals of time. Initially, when metal is hot and just poured in the mold it contains maximum amount of heat (heat content = super heat + heat of fusion) in it. As the time passes and metal begins to cool down in the mold it starts loosing its heat through different modes of heat transfer subsequently loosing its heat content [8,9]. Lost heat contents starts going to mold and mold temperature starts rising. The time up to which metal losses all its heat (both super heat and heat of fusion) is known as time of solidification, which could be clearly marked on the graphs, which again is a validity of the model.

5.4. Effect on rate of heat transfer

Graph shown in Fig. 6 is formed as a result of plotting data of rate of heat transfer against time in the process of solidification. It shows that rate of heat transferred is maximum in the beginning when metal is in its hottest state and the difference of temperature between outer and inner wall of mold is maximum. As the metal is poured in the mold it starts loosing its heat, which is transferred to mold and mold walls starts heating. With the passage of time rate of heat transfer across walls decreases because walls become hot and effective rate of heat transfer decreases as well. At a certain point when metal has lost all its superheat the rate of heat transfer becomes constant and does not change with the change of time. This is steady state of heat transfer [12]. During this step metal looses all its heat of fusion finally reaching its complete solid state.

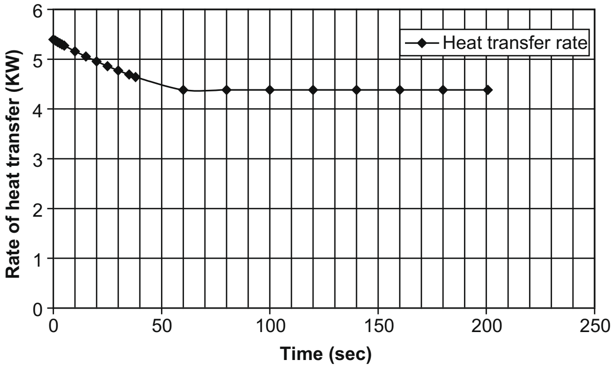

Fig. 6. Effect of time on rate of heat transferred from metal to mold and surroundings.

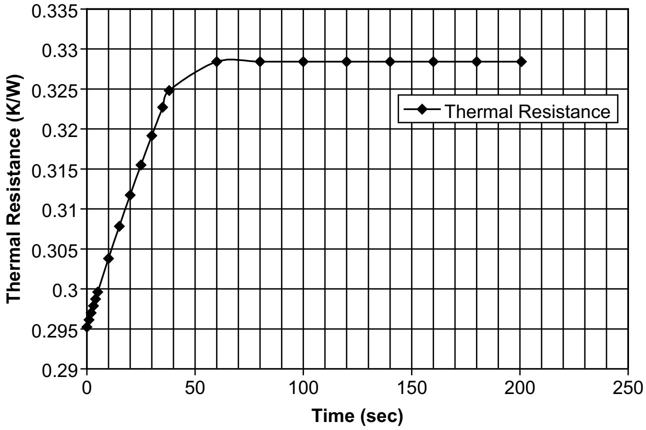

Fig. 7. Effect of time on the thermal resistance of mold material with decreasing temperature.

5.5. Effect on thermal resistance of mold material

When thermal resistance of mold material is plotted against time with change of temperature Graph shown in Fig. 7 is formed. This graph is an interesting graph, which shows the behavior of mold material with change of time with decreasing temperature. Mold materials initially have low thermal resistance but as time passes and more and more heat is transferred to mold and it gets heated, its thermal resistance starts increasing. Thermal resistance increases up to a certain point at which it becomes constant again. This is the time when arrest point occurs during cooling of metal and metal start loosing its heat of fusion. Thermal resistance remains constant as long as metal losses all its heat of fusion after which it again starts increasing (not shown) [12]. Through very small in amount this increase in thermal resistance has significant effect on the heat transfer capability of mold material. Mold starts offering more resistance to flow of heat as its temperature rises thus prolonging time of solidification. This is the reason why, cast iron molds are sometime sprayed with water from outside in order to keep them cool and keep their thermal resistance at minimum so that better heat transfer could be achieved [9,10]. But incase of ceramic mold increase in mold's thermal resistance in inevitable and cannot be avoided unless chills are used inside the mold to cause rapid cooling.

6. Conclusions

Following conclusions are drawn from experimentation and simulation of heat transfer phenomena during solidification of metal in investment casting:

(a) There is a significant effect of metal temperature (super heat), initial mold temperature and ambient temperature on the heat transfer pattern of castings.
(b) The accurate determination of HTC is the basis of accurate modeling and simulation of heat transfer phenomena.
(c) Effect of changed HTC due to airflow pattern and heated mold surface should be taken into account while modeling heat transfer phenomena in casting. HTCs are a strong function of temperature.
(d) There is a significant effect of change of mold materials properties on final heat transfer pattern, which should be taken into account while modeling the heat transfer phenomena. (Mold materials properties especially thermal resistance significantly changes with change of temperature.)
(e) Heat transfer patterns are very much dependent upon mold geometry as well configuration of parts attached to tree. They are also dependent upon mold wall thickness and number of coats and type of sand. (Higher wall thickness retards heat transfer.)
(f) Time of solidification is also a strong function of type and quantity of metal cast. A metal of high melting temperature can have longer solidification time similarly quantity of metal bears impact upon total quantity of heat released from metal which has to be transferred across wall to achieve solidification. Thus larger castings take longer time to solidify.

All the above mentioned factors bears an important affect on final time of solidification of investment casting in a mold and should be taken into account while designing investment casting process as well as modeling and simulating heat transfer phenomena during investment casting.

References

[1] J. Gebelin, M.R. Jolly, Modeling of the investment casting process, J. Materials Process. Technol. 135 (2003) 291–300.

[2] R.C. Givler, D.B. Saylors, Efficient runner networks for investment castings, Int. J. Numer. Methods Eng. 48 (2000) 1601–1614.

[3] G.K. Upaghya, S. Das, U. Chandra, A.J. Paul, Modeling the investment casting process: a novel approach for view factor calculations and defect prediction, J. Appl. Math. Model. 19 (1995) 354–362.

[4] L.V. Roches, J.C. Chevrier, Radiation Heat Transfer Modeling in Precision Investment Casting Technology, Solidification Processing 1987, The Institute of Metals, London, 1988. pp. 522–524.

[5] J.T. Anderson, D.T. Gethin, R.W. Lewis, Experimental investigation and numerical simulation in investment casting, Int. J. Cast Metals Res. 9 (1997) 285–293.

[6] A.S. Sabau, S.Viswanathan, Thermophysical properties of zircon and fused silica-based shells used in the investment casting process, Transactions of the American Foundry Society, vol. 112, Paper No. 04-081, 2004.

[8] D.R. Poirior, E.J. Poirier, Heat Transfer Fundamentals for Metal Casting, TMS Publication, USA, 1994. pp. 16–36.

[9] H.F. Taylor, M.C. Flemings, J. Wulff, Foundry Engineering, John Wiley and Sons, Inc., New York, NY, 1959. pp. 53–56, 74–80.

[10] R.W. Heine, C.R. Loper, P.C. Rosenthal, Principles of Metal Casting, McGraw-Hill Inc., New York, 1967 (Chapters 3 and 8).

[11] F.P. Incropera, D.P. DeWitt, Fundamentals of Heat and Mass Transfer, third ed., John Wiley & Sons, New York, NY, 1990.

[12] J.P. Holman, Heat Transfer, eighth ed., McGraw-Hill, New York, NY, 1997 (Chap. 2, 5 and 8).

[13] Castings: Metals Handbook, vol. 15, nineth ed., ASM International, Metals Park, OH, 1988.

[14] R.D. Pehlke, A. Jeyarajan, H. Wada, Summary of thermal properties of castings alloys & mold materials, NSF and Applied Research Division USA (1982) 143–160.